stochastic sec 4

Density Function $P(x)$ or $f(x)$

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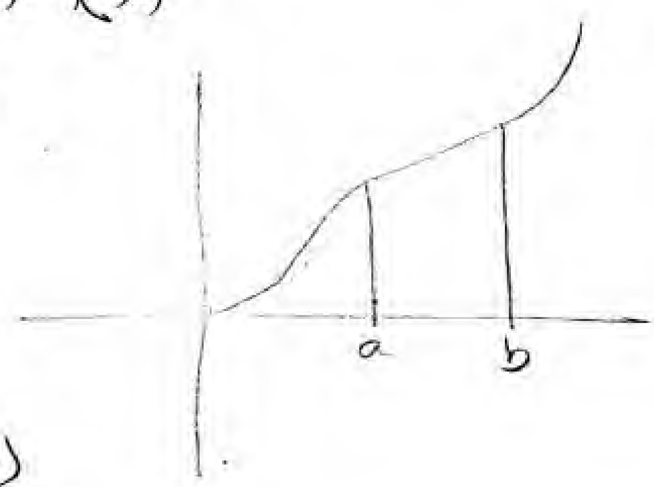
① $0 \leqslant f(x) \leqslant 1$ Probability

② $\int_{-\infty}^{\infty} f(x)\,dx = 1 \xrightarrow{x}$ Cont. Random Variable

$\sum_{-\infty}^{\infty} f(x) = 1 \longrightarrow x$ is a discrete

Cummutative fn $F(x)$

$F(x) = P(X \leqslant x)$



at most 3

$F(3) = P(x \leqslant 3)$

$= P(3) + P(2) + P(1)$

① $F(-\infty) = 0$

② $F(\infty) = 1$

③ $F(a) < F(b)$ if $a < b$

$\Big\} \longrightarrow$ increasing Function

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For density function

$$f(x) = \frac{dF(x)}{dx} \quad , \quad F(x) = \int_{-\infty}^{\infty} f(x)\,dx$$

sheet 4

① Let $f(x) = \begin{cases} x & 0 \le x \le 1 \\ \frac{3-x}{4} & 1 \le x \le 3 \\ 0 & \text{elsewhere} \end{cases}$

Prove that $f(x)$ is a density fn., then find $F(x)$

sol

You need to prove that $\int_{-\infty}^{\infty} f(x)\,dx = 1$

$$\int_{-\infty}^{0} 0 + \int_{0}^{1} x\,dx + \int_{1}^{3} \frac{3-x}{4} + \int_{3}^{\infty} 0\,dx$$

$$= \frac{x^2}{2}\Big|_0^1 + \left(\frac{3x}{4} - \frac{x^2}{8}\right)\Big|_1^3 = 1$$

$\therefore f(x) \longrightarrow$ density function

$$F(x) = \int_{-\infty}^{\infty} f(x)\, dx$$

for $-\infty \leq x \leq 0$

$$F(x) = F(-\infty) + \int_{-\infty}^{\infty} 0\, dx = 0$$

for $0 \leq x \leq 1$

$$F(x) = F(0) + \int_{0}^{x} x\, dx$$

$$= 0 + \frac{x^2}{2} = \frac{x^2}{2}$$

for $1 \leq x \leq 3$

$$F(x) = F(1) + \int_{1}^{x} \left(\frac{3}{4} - \frac{x}{4}\right)$$

$$= \frac{1}{2} + \left(\frac{3x}{4} - \frac{x^2}{8}\right) - \left(\frac{3}{4} - \frac{1}{8}\right)$$

$$= \frac{3x}{4} - \frac{x^2}{8} - \frac{1}{8}$$

for $3 \leq x \leq \infty$

$$F(x) = F(3) + \int_{3}^{x} f(x)\, dx = 1$$

$$F(x) = \begin{cases} 0 & -\infty \leq x \leq 0 \\ \frac{x^2}{2} & 0 \leq x \leq 1 \\ \frac{3x}{4} - \frac{x^2}{8} - \frac{1}{8} & 1 \leq x \leq 3 \\ 1 & 3 \leq x \leq \infty \end{cases}$$

$\boxed{2}$ If $f(x) = \frac{1}{2^x}$ for $x = 1, 2, 3, \cdots$

Can $f(x) \rightarrow$ Probability $f\underline{n}$.

sol

$$\sum_{-\infty}^{\infty} f(x) = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \cdots$$

$\hookrightarrow = 1 \longrightarrow$ density

$\boxed{4}$

$$\text{If } f(x) = \begin{cases} K(2-x) & 0 \leqslant x \leqslant 2 \\ 0 & \text{elsewhere} \end{cases}$$

Probability function $\quad K = ?$

$$\therefore \int_{-\infty}^{\infty} f(x)\, dX = 1$$

$$\int_{0}^{2} (2K - Kx)\, dx = 1$$

$$2KX + \frac{Kx^2}{2} = 1 \Rightarrow 4K - 2K = 1$$

$$\boxed{K = \frac{1}{2}}$$

$\boxed{5 \rightarrow \text{the same idea}}$